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Ejercicios para la enseñanza-aprendizaje de las razones trigonométricas
Exercises for the teaching-learning of trigonometric ratios

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Resumen

En este artículo se ofrece una propuesta de ejercicios como recurso didáctico para perfeccionar el proceso de enseñanza-aprendizaje de las razones trigonométricas en el 9º grado del nivel Educativo Secundario, contenido que se ha convertido en un proceso memorístico y rutinario, sin ningún sentido ni utilidad para los educandos. Para dar cumplimiento al objetivo se utilizaron métodos del nivel teórico como inductivos-deductivos, análisis-síntesis y revisión documental que permitieron recopilar la información necesaria y elaborar la propuesta.

Palabras clave: Ejercicios didácticos; Recursos didácticos; Funciones didácticas; Proceso de enseñanza-aprendizaje; Razones trigonométricas

Abstract

This article offers a proposal of exercises as a didactic resource to improve the teaching-learning process of trigonometric ratios in the 9th grade of Secondary Education, a content that has become a memorized and routine process, without any sense or usefulness for the students. In order to fulfill the objective, theoretical methods such as inductive-deductive, analysis-synthesis and documentary review were used to gather the necessary information and elaborate the proposal.

Keywords: Didactic exercises; Didactic resources; Didactic functions; Teaching-learning process; Trigonometric ratios

Introduction

Trigonometry is a part of mathematics that is difficult for students to understand. This difficulty in understanding this topic could be influenced by the different ways of understanding and representing its basic notions, the ways of approaching it, notions such as the trigonometric circumference, right triangles or trigonometric functions. Despite all this, very little has been done to investigate the didactic complexity underlying trigonometry. However, other topics related to trigonometry have received more attention, such as the study of the understanding of trigonometric functions and the history of trigonometry education. Examples of such research on these topics are by Weber (2005), Dominic (2012), Allen (1977), and Sickle (2011), respectively. In addition, the need for research to facilitate students to overcome difficulties encountered during the teaching-learning process associated with trigonometric ratios has led some researchers to make innovative didactic proposals (Blackett and Tall, 1991; Weber, 2005; Kutluka, Furkan and Zengin, 2011). Other research in this direction includes Kendal and Stacey (1997), Barnes (1999), Miller (2001) and Searl (1998).

Desarrollo

The teaching of trigonometric ratios is traditionally based on the presentation of algorithms and formulas that are taught to students, to manipulate objects that they do not know what they mean, to find answers to problems they do not understand. The trigonometric ratios in the right triangle constitute the fundamental basis of the other concepts in trigonometry, since it is evident that a poor understanding of these ratios produces difficulties in the learning of more complex concepts of trigonometry.

Therefore, a system of didactic exercises is proposed as an objective to contribute to the teaching-learning of trigonometric ratios in 9th grade students of the Secondary Education level.

Exercises can be considered as any human activity aimed at developing or preserving a psychic faculty or quality, so that in the learning of different disciplines, they serve as a complement and verification of theoretical teaching.

The concept of exercise and their usefulness in the teaching-learning process of mathematics have been addressed by various specialists, among which stand out: Jung (1981), Müller, (1987), Ballester, Santana, Hernández, Cruz, Arango, García, et al. (1992), Ballester & Jon (2011). According to Ballester et al. (1992), the concept of exercise in the teaching of mathematics has been defined by several specialists, most of whom agree in defining it as: "[...] a requirement for the performance of actions, solution of situations, deduction of relationships, calculation" (p. 406).

Müller (1987) defines exercises in mathematics teaching as a requirement to act, characterized by three elements:

- The objective of the actions: resides in transforming an initial situation (the given or known) into a final situation (the sought or unknown).
- The content of the actions: may be given by the elements of the mathematical subject to which they relate (concepts, propositions, algorithms) and by the type of action (identify, compare, order, classify, substantiate and control).
- The conditions of the actions: reside in the demands posed by the exercise, expressed by the degree of difficulty of the exercise.

In this regard, Ballester (1992) points out:

an exercise is a requirement that propitiates the realization of actions (...) each action must specify the objective that moves us to transform the premise to obtain the thesis; the content that includes the types of actions (identify, compare, classify, classify, substantiate, etc.), the object of the actions (concepts, propositions, algorithmic procedures), the correspondence between situations and heuristic procedures and auxiliary heuristic means. (p. 459)

There are also multiple classifications of the types of exercises. For example, Jung (1981), elaborated a classification of exercises based on the degree of abstraction in the reflection of elements and relations, thus the type of reflection that is performed. He assumed mathematical exercises as a reference, subdividing it into two subordinate concepts: (Image 1)

- Exercises of application, those that have their origin in practice.

- Constructed exercises, these in turn undergo another division; the formal ones, within them he mentions (solving an equation, solving a system of equations) and on the other hand, exercises with texts made up of those whose text is purely mathematical or are related to practice.

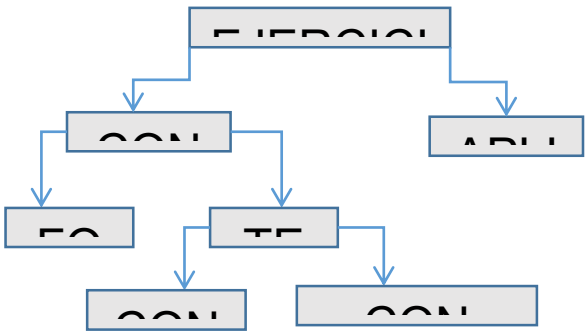


Image 1. Classification of mathematical exercises
Source: Taken from Jung (1981)

In the field of Pedagogy and especially in Didactics as a pedagogical discipline that investigates and elaborates the most general principles and whose object of study is the teaching-learning process, we find a first problem. Can exercises be considered as didactic resources? Undoubtedly, it is an aspect to reflect on that we will address later, from this perspective.

Chancusig (2017), states that didactic resources are the means to which one can resort as an alternative during the teaching-learning process in order to fulfill an objective favorably. These interactive didactic resources refer us directly to training; training and instruction for teaching, resources are tools that have utility within an educational process, making the use of a didactic resource would help the teacher to fulfill his educational function. In general, the resources provide information, serve to put into practice what has been learned and sometimes these materials are a guide for our students and we can use them as motivation for learning.

For Raffino (2020), didactic resources, didactic materials or didactic aids are any type of material or technological support that facilitates or facilitates the teaching-learning process. They are usually used by learners in pedagogical or training institutions as a way of complementing or making their work more efficient.

There is no strict and universal concept of what is and what is not a didactic resource. Basically because anything can be one, as long as it fulfills the function of facilitating learning or adapting it to the specific needs of a certain type of student.

At present, a didactic resource is considered to be any means, whether material or conceptual, that is used as a support to the teaching-learning process, which is generally face-to-face, with the purpose of facilitating or stimulating learning, for example: blackboard, overhead projector, transparencies, slides, video, computer or others, and whose essential characteristic is its incidence in the process of educational transmission and which

is also conceived in accordance with learning, forming part of educational communication, and can be classified as: oral, written and audio-visual.

Didactic resources are fundamental tools that complement the teaching-learning process. A teacher who does not use didactic resources in his classes is a teacher lacking in didactics and his students will not learn in the best way.

Therefore, it is worth highlighting the importance of didactic resources in the teaching-learning process of mathematics, so it could be said that without resources there is no learning.

For example, some resources will allow significant learning, with high student participation, while others will serve more as a communicative support for the teacher, or simply as reinforcement material.

García & Cruz (2014), consider that in order to create a didactic resource, the following functions should be kept in mind:

1. Motivational function: awakens interest in the topic or subject to maintain attention during the process of studying the content. They are tangible materials that can be manipulated by the student, motivate the learning process and can be used over and over again many times for different purposes.

2. Facilitating function: they expose clear terms that locate the exposure of the learners, providing them with information. They facilitate the strengthening of the educational process in order to enhance the quality of education.

3. Guidance function: fosters the capacity for organization and study. They are a guide for learning, since they help to organize the information we want to transmit. In this way we offer new knowledge to the student.

4. Developmental function: it favors learning processes of skills, knowledge, they help us to exercise skills and also to develop them.

5. Evaluation function: the didactic resources allow us to evaluate the knowledge of the learners at each moment, since they usually contain a series of questions on which we want the student to reflect, it performs in the student a general review of what has been learned and then evaluate in order to stimulate a deliberation on their adequate learning.

Types of didactic resources

Raffino, (2020) considers that didactic resources can be classified as follows:

- Permanent working material. Everything that is used daily in teaching, either to keep record of it, to illustrate what is said or to allow other types of operations.
- Informative material. Those materials in which information is contained and which are used as a source of knowledge.
- Illustrative material. All that which can be used to accompany, enhance and exemplify the content taught, whether visual, audiovisual or interactive.
- Experimental material. That which allows students to verify through practice and direct experimentation the knowledge imparted in class.
- Technological material. These are the electronic resources that allow the generation of contents, the massification of the same; using mainly the so-called ICT.

Then it can be questioned, if the exercises:

1. Can they be used to support the teaching-learning process?
2. Do they enhance, facilitate or stimulate the learning process?
3. Are they part of the instructional-educational communication process?
4. Can they be formulated in relation to the process itself in the assimilation of knowledge, habits or skills?

It is evident that on the basis of these theoretical precepts, exercises are didactic resources as long as they comply with these characteristics, meaning any exercise in accordance with certain objectives and whose function serves to support the learning process with the purpose of stimulating it and which constitutes part of the communication process, in an instructive-educational context and as a means of application of knowledge to consolidate it and at the same time, lay the foundations for the acquisition of new knowledge.

It is precisely from this perspective that a regulated change is sought in the quantity and qualification of supports, aids, strategies, ways, methodologies, didactic actions and resources for teaching-learning, which may involve aspects as diverse as the motivational-affective sphere, the management of attention processes, analytical memorization resources, learning induction and procedures for the efficient management of information.

Exercises as didactic resources are important ways if they are exploited in all their magnitude, but they must also comply with certain requirements, otherwise there is a risk of obtaining results inversely proportional to the expected ones. Therefore, a series of premises are necessary, it is important to emphasize the need to consider and comment on different ways of solution as a guiding basis of the activity that forces the students to reflect on the knowledge applied and the ways of carrying them out and that make possible the proposals of new solutions. Besides being ready to defend and argue their points of view in correspondence with the selected method, exercises should be proposed using different ways and reformulate the more complex ones, constituting in the first instance, an alternative aimed at stimulating the development of thinking and reflection.

When using exercises as didactic resources, it is very important to take into account that they should have a scientific character, linking theory with practice and the concrete with the abstract. They must be systemic and accessible, which will guarantee the solidity of the knowledge.

When using the exercises as didactic resources, the particularities and motivations of the students should be considered in order to achieve an active performance in the execution process and stimulate their reflective activity. The teacher should offer precise indications to students who do not achieve a first level of resolution according to the objectives and propose simpler exercises. No less important is the formulation of exercises, a task that stimulates the search for relationships and the establishment of dependencies, which requires reflection on the conditions, requirements and ways to solve them. To achieve this, it is necessary to proceed gradually and work with different solution alternatives.

Classification of didactic exercises

We assume the classification given by Batista (2017) on didactic exercises, which proposes a classification of exercises taking into consideration the different didactic functions: preparation for the new teaching subject, orientation towards the objective, treatment of the new subject, consolidation and control. Hence, a typology can be established with the exercises in their character of didactic resources, which can be classified according to their function in the different moments of the activity.

The different forms of teaching organization are governed by their system of didactic principles and functions, which guarantee the success of the teaching-learning process, since they organize the modes, actions, models or steps to be followed, influencing in a positive and organized way the practical activity of the learner.

As has been pointed out, each function has a specific objective within the teaching-learning process, allowing the organization in the assimilation of the teaching material. We will use the content on trigonometric ratios to exemplify in each of the didactic functions how to use the didactic exercises.

Exercises for the preparation of the new teaching subject.

They are all those exercises whose objective is to determine the mastery or not of premises or preconditions for the assimilation of new contents.

The previous knowledge required for the study of Trigonometric Ratios is:

- Angles and measures. Triangles and relationship between their sides and angles.
- Pythagorean Theorem.
- Numerical sets (rational and irrational).

- Solving equations of 1st and 2nd degree.

The study of trigonometric ratios requires knowledge of the angles and how they are represented mathematically, thus the elements of a right triangle, as they will constantly appear in the various activities and it is necessary to be familiar with them.

a) In addition, it will be necessary for students to remember the Pythagorean Theorem to use it in demonstrations and in solving some problems.
b) The Pythagorean Theorem is a fundamental relationship in Euclidean geometry between the three sides of a right triangle. It states that in every right triangle, the square of the hypotenuse (the side opposite the right angle) is equal to the sum of the squares of the other two sides called legs.(Wikipedia, 2021)
c) For example: to ensure the preconditions for the assimilation of trigonometric ratios in a right triangle, one can start from the following exercise:
d) Draw an acute angle and denote it by < ABC.
e) Draw three straight lines perpendicular to side $(BC)^{-}$ that intersect side $(AB)^{-}$

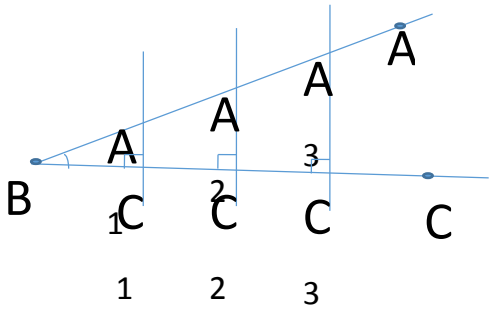


Figure 1. Similar Right Triangles
Source: Taken from Mathematics textbook 9th grade (2015).

As can be shown, the three triangles formed are similar because they have respectively two equal angles (the <ABC common to each triangle and the right angle).

Establish the equal ratios between the leg opposite the common angle <ABC and the hypotenuse in each triangle.

$(A_1 C_1)/(BA_1) = (A_2 C_2)/(BA_2) = (A_3 C_3)/(BA_3) = K1$

Set the equal ratios between the other leg (adjacent to <ABC) and the hypotenuse in each triangle.

$(BC_1)/(BA_1) = (BC_2)/(BA_2) = (BC_3)/(BA_3) = K2$

(e) Establish the equal ratios between the leg opposite the common angle <ABC and the other leg (adjacent to <ABC) in each triangle.

(A_1 C_1)/(BC_1) = (A_2 C_2)/(BC_2) = (A_3 C_3)/(BC_3) = K3

It can be seen that for the same acute angle ABC the ratio between two sides of one of the formed right triangles and the ratios of their homologous sides in the others are equal. Therefore, their result does not depend on the lengths of the sides of the triangles.

However, if we vary the amplitude of the <ABC and do the above analysis, we find that the ratios considered are unequal.

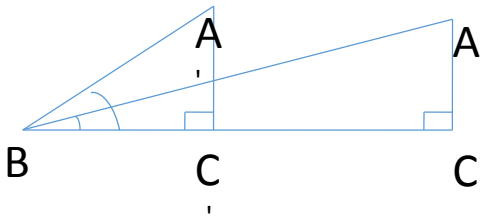


Figure 2.Non-similar right triangles

Source: Taken from Mathematics textbook 9th grade(2015).

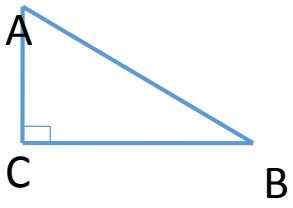
Indeed, if the amplitudes of the angles A' B C and ABC are different, then the triangles A' B C ' and ABC are not similar and the considered ratios are unequal.

Goal-oriented exercises

The expectation in this type of exercise is to direct the learners' attention to the object of knowledge, what they are going to know, how they are going to do it and the importance of knowing what is new. It is evident that if this type of exercise is done with the utmost rigor, it should awaken interest in the learner towards the object of study.

For example.

We could start the motivation by analyzing that in a right triangle ABC, one side can be determined knowing the other two, if the Pythagorean theorem is applied.



$AB^2 = AC^2 + BC^2$

$< A + < B + < C = 180°$

Figure 3. Right triangle ABC

Source: Own elaboration

Likewise, given two angles of any triangle, the third angle is calculated using the property of the sum of the interior angles of a triangle.

By analogy, it is concluded that given two elements of a right triangle we can determine the third. Now, if we vary the conditions we find the following problem: in a right triangle, could we find the length of a known side, another side and an acute angle? or could we calculate the amplitude of an angle knowing two sides?

From these questions, we orient the study of trigonometric ratios in a right triangle. Trigonometric ratios are tools that allow us to solve problems of calculating distances between two points in conditions where other methods of measurement cannot be used, because we find that one or both of the points are inaccessible or because there is an insurmountable obstacle between them.

We could also use the following situations to motivate and guide the study of trigonometric ratios, related to our reality, to awaken the interest of students.

a) The heights (in relation to sea level) at which two points A and B are located are, respectively, 812 m and 1020m. From point A point B is seen at an angle of 30° with the horizontal plane, as shown in the figure. Determine the distance between A and B.

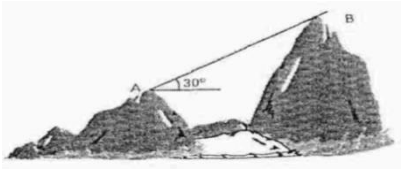


Figure 4. Distance between two inaccessible points

Source. Own elaboration

Exercises for the treatment of new material

These are the exercises whose objective is to condition the discovery by the students of the essential characteristics of the concepts, judgments and the mechanisms of acquisition of new ones. Following the

previous exercise that serves as a preparation for the assimilation of the content on trigonometric ratios, we can treat this subject in the following way:

Elaborating the definitions of the trigonometric ratios of an acute angle in a right triangle and knowing that:

Ratio in mathematics means a division, and metric means measure, therefore the phrase trigonometric ratios can be associated to the division of the measures of the sides of a triangle.

We can retake the ratios established according to figure 1

$$\therefore \frac{A_1C_1}{BA_1} = \frac{A_2C_2}{BA_2} = \frac{A_3C_3}{BA_3} = K_1$$

$$\frac{BC_1}{BA_1} = \frac{BC_2}{BA_2} = \frac{BC_3}{BA_3} = K_2$$

$$\frac{A_1C_1}{BC_1} = \frac{A_2C_2}{BC_2} = \frac{A_3C_3}{BC_3} = K_3$$

Define a:

K1 = Sine of angle ABC

K2 = Cosine of angle ABC

K3 = Tangent of angle ABC

To designate the sine, cosine and tangent of angle B, we will use the following notation: sin B, cos B and tag B respectively. The following exercise is then proposed to the students:

In the right triangle ACB in Figure 5:

(a) Identify the opposite leg, the leg adjacent to angle A and the opposite leg, the leg adjacent to angle B.

b) Establish the trigonometric ratios for each acute angle.

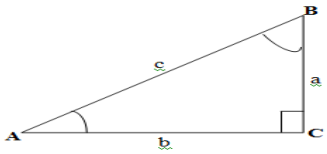


Figure 5. Right triangle ACB

Source: Own elaboration

Consolidation exercises

The objective of this type of exercise is to concretize the product of its generalizations in particular cases, having to reconsider the essential characteristics previously assimilated.

a) How to calculate the height of the Victory Day School Center, if a student stands 30 m away from the base of the school and observes the height of the building at an angle of 30°.

b)

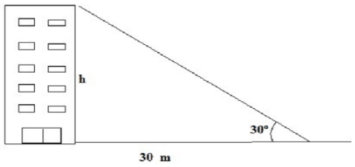


Figure 6. Height of a school

Source. Own elaboration

b) When the sun is at 45° above the horizon, a building 100 m high casts a shadow of measure x. What is the result of x in meters?

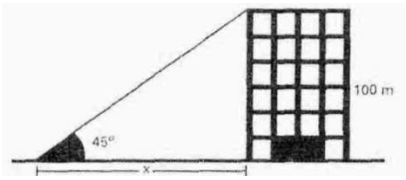


Figure 7. Shade cast by a building

Source. Own elaboration

a) A 12 meter long ladder (BC) is supported on top of a wall. Knowing that the distance from the wall to the foot of the ladder is 6 meters (AB), determine the angle of inclination of the ladder in relation to the wall.

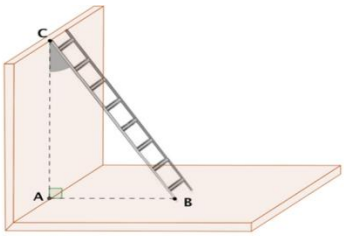


Figure 8. Angle of inclination of a staircase

Source. Own elaboration

Control exercises

It is any exercise whose objective is to determine the degree to which the object of knowledge has been assimilated. There is no better evidence for these types of exercises as didactic resources, although there are many others, than those used in exams, tests or quizzes, which allow to control the achievement of the proposed objectives.

As can be seen in each type of exercise as a didactic resource in the classification proposed above, the essential or distinctive characteristics are the objectives or functions they fulfill in the different moments of the teaching-learning process. In all types, they are part of the educational communication, support the learning process with the purpose of stimulating it and are means of applying knowledge to consolidate it and lay the foundations for the acquisition of new knowledge.

Conclusions

The elaboration of exercises as didactic resources entails eliminating improvisation, anarchism and unnecessary repetition, since theory and practice, abstract and concrete, systematicity and accessibility must be scientifically linked to guarantee the achievement of the objectives set.

If the classification of the exercises as didactic resources is instrumented starting from the function in the different moments of the class, it provides a safe and effective way to achieve the success of learning and the development of skills in the resolution of the problems presented in the content on trigonometric ratios.

With the classification proposal, we do not intend to exhaust and much less to put an end to the problem, but to stimulate the debate thus contributing to improve this problem that so far does not augur a quick consensus. It remains open for debate, reflection, criticism and, why not, suggestions.

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