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Actividades para consolidar las habilidades sobre el cálculo mental en los educandos de sexto grado

Activities to consolidate mental arithmetic skills in sixth grade students

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Resumen

El objetivo del presente trabajo es socializar actividades para consolidar el cálculo mental en los educandos de 6to grado, partiendo de insuficiencias que estos presentan en la aplicación de estrategias para calcular mentalmente contenidos básicos para el cálculo escrito. Se aplicaron métodos científicos del nivel teórico y empírico que permitieron determinar las dificultades y elaborar la propuesta. Entre los resultados más importantes se destacan las estrategias para el cálculo mental con números naturales y racionales no negativos y la propuesta de actividades para el desarrollo de estas estrategias.

Palabras clave: Actividades; Habilidades; Cálculo mental; Proceso de enseñanza- aprendizaje.

Abstract

The objective of this work is to socialize activities to consolidate mental arithmetic in 6th grade students, based on their insufficiencies in the application of strategies to mentally calculate basic contents for written arithmetic. Theoretical and empirical scientific methods were applied to determine the difficulties and elaborate the proposal. Among the most important results, the strategies for mental calculation with non-negative natural and rational numbers and the proposal of activities for the development of these strategies stand out.

Keywords: Activities; Skills; Mental calculation; Teaching-learning process.

Introduction

Traditionally, the teaching of mental arithmetic has emphasized the repeated practice of operations in order to solve them as quickly as possible (in the head), without the necessity of using pencil and paper. However, this vision is not entirely complete, since having mental arithmetic skills means more than just accumulating a series of isolated numerical facts in memory. On the contrary, to be agile in computation one must be able to interconnect, understand and master a large number of ideas and concepts. In other words, the ability to calculate does not depend so much on a good store of isolated facts, operations or results, as on a good numerical sense. Thus, it would be more correct to conceive of mental calculation as the invention and application of strategies based on the characteristics of the number system and arithmetic operations.

Several researchers on the subject, such as Ponte & Sarracina (2000), Bourdenet, Caney and Watson (2003), agree on the need to work on mental arithmetic in the classroom, since it provides the student with an opening to new ways of thinking and mental agility that will help him/her to solve problems in a more competitive way. These researchers denounce the neglect of mental arithmetic in Primary Education classrooms, the scarce treatment of it in textbooks, and the deficient instruction that, in general, takes place in Teacher Training.

The authors of this paper consider that mental arithmetic contributes to the acquisition of other important skills for learning mathematics. In this perspective, mental arithmetic develops in student's notions of order and logic, reflection and memory, contributing to their intellectual formation and providing them with tools to perform simple calculations without the aid of written arithmetic and thus preparing them for everyday life.

In interviews with students and teachers, it was found that there are difficulties to calculate mentally and that this is due to the fact that this type of operation does not constitute a systematic practice in the classroom, in addition, the classes on the properties of arithmetic operations were given formally, often without observing whether the objectives to be achieved were attained. It was evidenced that teachers do not dedicate enough time to mental arithmetic, recommending calculating machines so that the process of solving exercises is as fast as possible.

Therefore, activities to consolidate mental arithmetic in 6th grade students are proposed as an objective.

Development

Mental calculation and its importance

Mental arithmetic is an important tool these days when it comes to calculating with money, time, mass, distances. Calculation skills are essential to maintain a solid relationship with numbers so that we can look at them critically and interpret them properly. In this sense, mental arithmetic is a crucial and effective element that the learner must know to use with confidence.

According to Ponte & Sarracina (2000), in everyday life, most of the calculations we do are mental. Paper and pencil cannot always be used, nor is it necessary. In many situations, the answer does not have to be precise, but an approximation is sufficient.

When you need to get exact results that cannot be reached with mental calculations, you can use technology. Even when using a calculator, it is good to estimate the result first so that an error can be detected while pressing the keys.

As Ponte and Sarracina (2000) express, the development of mental arithmetic cannot be understood without also developing number sense, since promoting the use of appropriate methods for calculating in students helps to develop number sense and mental arithmetic strategies.

Developing mental arithmetic skills in learners is not an easy task and requires intention, method and persistence. Teaching mental arithmetic without method is of little use, and it is a complement to written arithmetic and should be taught methodically and regularly, with frequent but brief lessons to maintain arithmetic skills. As a main objective, mental arithmetic aims to improve the practice of the four arithmetic operations, getting used to operate with large numbers quickly and safely.

According to Bourdenet (2007), working with mental arithmetic regularly allows the learner to be more flexible in changing number recognition. For example, in the operation 25 × 0.25, consider 1/4 instead of 0.25. He also states that mental arithmetic moments in the classroom compare, reflect, reflect, reflect, conjecture, analyze errors, and promote intense discussion fundamental to making connections between mathematical learning. This author also emphasizes the importance of discussing the calculation and the error with the whole class as a way of learning, since the moment of correction repeated regularly and considering different possible procedures promotes significant learning and allows a solid knowledge.

However, the social interaction triggered during mental arithmetic classes favors individual and collective learning. From the individual point of view, it helps the learner, on the one hand, to organize his thinking, because he has to express it to others, increasing the degree of articulation and precision in verbalization. On the other hand, it facilitates cognitive work, since the learner is encouraged to quickly find a solution to the problem presented, looking for effective and appropriate techniques, as well as leading him/her to explore other ways.

Arguments for the development of mental arithmetic

We all need mental arithmetic in daily life and as such, we should have an idea of what it involves. Due to the introduction of the calculator, mental arithmetic has gained increasing importance in the teaching of arithmetic; it is a concept adopted by a group of mathematical teachers and researchers and has gained international consensus. Briefly, it can be said that this concept consists of active, flexible and skillful arithmetic calculation, whereby:

(a) It allows everyone to choose his own method.

b) It can be adapted to the numbers in question.

c) It requires understanding and can only be used if it is understood.

Mental arithmetic, as a powerful means of calculation, is fundamentally a form of approximation to numbers and numerical information. It is an elementary competence characterized by:

1. Working with numbers and not digits.

2. Using the elementary properties of calculus and the relationship between numbers such as commutative property, distributive property and the notion of inverse operation.

3. Involves a well-developed number sense and a healthy knowledge of basic number facts.

4. Allow the use of intermediate registers according to the situation.

For the authors of this paper, mental calculation has three elementary skills that, analyzed from a learning point of view, interact with each other and their acquisition is accompanied by a broader understanding of numbers and operations; they are:

- Calculation where numbers are first seen as objects on a counting line and where operations move along the line: forward (+), backward (-) or repeatedly forward (×), or repeatedly backward (÷).
- That numbers are preferably viewed as objects, as a decimal structure and where operations are performed by decomposing numbers based on this structure.
- Calculation based on arithmetic properties where numbers are viewed as objects that can be structured in various ways and where operations are performed using the appropriate properties.

Each of these basic forms can be used to varying degrees. To a lesser degree by using models such as the empty line or money, and to a greater degree by recording intermediate degrees in arithmetic language or simply calculating mentally. These basic forms can be introduced and practiced as extensions of each other.

Process of teaching and learning mental arithmetic

Essential to the acquisition of numeracy skills is a process of inquiry into numbers within different domains and the development of strategies with which to explore andprogressively teach the basic forms.

Beginning with a number inquiry, such as:

- Researching partitioning strategies that flow naturally into number inquiry and those learners, under teacher guidance, can build on their own.
- Extending this process to decomposition strategies (which some learners may have already discovered at earlier stages) when learners are already sufficiently confident and as a result, their understanding of numbers and the relationships between them has increased significantly.

- The process can be extended to varied compensation strategies when learners have sufficient confidence with the previous strategy and their understanding of operations is deepened.

This means that the learner cannot use varied strategies much earlier, but that the emphasis in teaching should start with partitioning strategies; it is only when the learner has mastered this strategy perfectly that decomposition strategies and in more advanced stages various strategies should follow. If the order in the learning process is not correct and deep enough, there is a danger that learners with difficulties will get lost and not understand the various types of approach.

The collective discussion of the various types of strategies that the learner develops on what they observe helps them to appropriate a repertoire of strategies with their own limits and flexibility, it also teaches them how to decide which one to use. The greater the development of mental calculation skills, the more comfortable the learner will be in using standardized calculation strategies, such as algorithms. If mental arithmetic is practiced through regular short-term activities, even the most difficult learners can make progress by becoming more proficient in mental arithmetic. Generally, one begins by studying and practicing mental arithmetic with operations up to 100.

Learners have first contact with addition and subtraction, then with multiplication and division, however, it should be remembered that many calculation skills are consolidated with the relationship established between the various operations. For example, multiplication is the inverse of division and is also the result of successive additions.

Mental arithmetic should be present in the classroom every day. Performing five calculations at the beginning of each class to be solved in 5 minutes is enough to systematically guide learners towards appropriate calculation strategies.

Besides being able to dedicate a specific moment of the class to the development of mental arithmetic strategies, it is important not to forget that the whole class is a favorable context for the development of mental arithmetic where the teacher has an important role in its integration and in problem solving at times when it becomes faster than calculating by the usual algorithm or can help learners to criticize a result or in an approximate calculation.

Mental calculation strategies

As Ribeiro (2009) points out, mental arithmetic strategies, when known, understood and applied, allow effective and fast calculation. Although mental arithmetic allows the use of personal strategies, there are a number of strategies that should be taught, discussed and trained with learners.

Mental arithmetic strategies for use with natural numbers and for the four operations:

I. Decomposition of numbers: strategy used in all four operations. For example:

(a) In addition and subtraction operate order by order.

235 + 462 = 200 + 400 = 600;

30 + 60 = 90;

5 + 2 = 7;

600 + 90 + 7 = 697

a) In multiplication, it decomposes the product into several products.

4 × 15 = 2 × (2 × 15) = 2 x 30 = 60

b) In division, factor the divisor into several equal factors.

249 ÷ 3 = 240 ÷ 3 + 9 ÷ 3 = 83

I. Compensation: strategy used for addition and subtraction where, for example, one adds and/or subtracts a close number and the result subtracts the one that was added the most or the one that was added the least.

478 + 98 = 478 + 100 - 2 = 578 - 2 = 576

II. Use of properties of operations: a strategy involving the use of inverse operations, commutative and associative properties in addition and multiplication, distributive in multiplication.

Applying the commutative property a + b = b + a, is usually simpler (faster and more often successful) the sums in which the first addend is greater than the second. So, especially in sums with numbers greater than ten, it may be convenient to add the smallest to the largest.

7 + 21 = 21 + 7 = 28

13 + 54 = 54 + 13 = 67

For three or more addends, this property allows us to regroup the quantities to make the sums simpler.

35 + 24 + 5 = (35 + 5) + 24 = 40 + 24 = 64

Reduction to sum. In different situations, it is important not to forget that a multiplication is a sum of equal factors.

215 - 2 = 215 + 215 = 430

Using the distributive property means decomposing a factor into additions or subtractions (looking for rounding) and then applying the distributive property:

82 - 7 = (80 + 2) - 7 = 560 + 14 = 574

39 - 4 = (40 - 1) - 4 = 160 - 4 = 156

42 - 12 = 42 - (10 + 2) = 420 + 84 = 504

To multiply mentally a number by a factor digit 27 - 8, we start by multiplying not the units, as in the written calculation, but the tens of the multiplicand (20 - 8 = 160), then multiply the units (7 - 8 = 56) and then add both results (160 + 56 = 216).

Factorization: consisting of decomposing one or both factors into simpler ones, not necessarily prime. Its structural basis is the associative property of multiplication, but occasionally, the commutative property is used.

18 - 15 = 2 - 9 - 5 - 3 = 10 - 27 = 270

Mental arithmetic with non-negative rational numbers

Wolman (2006) states that mental arithmetic with fractions and decimal numbers can be developed daily when learners compare fractions and/or decimals, work with equivalent fractions and perform operations.

Caney and Watson (2003) studied mental calculation strategies with rational numbers for learners. These authors emphasize the importance of understanding the relationship between different representations of a rational number in order to develop mental calculations with rational numbers. In this study, some of the strategies used by learners use a previously memorized rule and sequentially place a combination of strategies, such as converting decimals into fractions to build the whole.

These authors refer to ten strategies used by learners: change of operation, change of representation, use of equivalences, use of known facts, repetition of the addition/multiplication operation, establishment of connections, working with parts of a second number, working from left to right, use of mental images and use of memorized rules.

Change of operation: this strategy consists of transitioning between inverse operations, change of representation, use of different representations of a rational number (fraction, decimal, percent) or whole numbers for 10/100 where, for example, in the operation 0.19 + 0.1 is taken as 0.19 as 19 and 0.1 as 10.

Use of known facts: learners make some correspondences with what they already know. For example, when calculating 10% of 45, they use the knowledge they have about 10% to get first 10% of 40 and then 10% of 5.

Repeat operations: learners do successive addition / multiplication or use doubles and halves. To calculate 4 × ¾, multiply the fraction twice and again twice and in calculating 25% of 80, calculate half of 80 and then again half of the previous half.

Working with parts of a second number: learners use several strategies. To calculate 10% of 45, do divisions by place value, dividing 40 by 10 and then 5 by 10 or dividing numbers into parts where 0.5 + 0.75 can be seen as 0.5 + 0.5 + 0.5 +0.25.

Work from left to right: work first with the whole part and then with the decimal part (4.5-3.3 calculation 4-3 = 1 and then 0.5-0.3 = 0.2) or divide the number by the place value only after the decimal point, worked first with the tenths and then with the hundredths.

Use of mental images: learners mentally construct pictorial representations especially of fractions and operate by adding or deleting parts or use mental forms of algorithms in which they operate by mentally visualizing the algorithm.

Mobilization of memorized rules: learners use previously memorized calculation rules and quickly apply a calculation procedure. For example, to perform 1.2 × 10, simply move the comma one square to the right.

How are commutative, associative, and distributive properties useful in the mental computation of multiplication and division?

Consider the following calculation: 5x28.

- For some it may be easier to do 28 times 5, because it is easier to multiply by 5. It is a commutative property that allows you to change the order of the two numbers in multiplication, i.e.: 28 x 5 = 5 x 28.
- To calculate 28 x 5, we can consider 28 as 14 x 2, first do 2 x 5 (to get 10) and then multiply 14 by 10 (14 x 10 = 140). What is used here is the associative property, i.e. (14 x 2) x 5 = 14 x (2 x 5).
- Another alternative to calculate 28 x 5 would be:

28 = 20 + 8

20 x 5 = 100 and 8 x 5=40, whose sum is 140.

We are using the distributive property of multiplication with addition:

5 x (20 + 8) = (5 x 20) + (5 x 8).

- Another hypothesis would be to think of 28 as 30 - 2 and then multiply by 5.

5 x 30 =150 and 5 x 2 =10. Then 150 - 10 =140.

The multiplicative distributive property with respect to subtraction is being used:

5 x (30 - 2) = (5 x 30) - (5 x 2).

Although there are no commutative properties in division, there is distributivity in relation to addition and subtraction, properties that are used daily.

We simplify the calculations we do mentally. To do this, we have to look for numbers that are easy to relate to a particular divisor.

- For example, to calculate 143: 11, we express 143 =99 + 44 and then 143: 11 = (99: 11) + (44: 11) = 9 + 14 = 13.
- For example, to calculate 162: 9 we express 162 = 180-18 and then (180-18): 9 = (180: 9) - (18: 9) = 20-2 = 18

The most common strategies are the use of known facts that include lessons learned in dealing with non-negative rational numbers, such as knowledge of learners' strategies and errors in mental computation with non-negative rational numbers. The use of inverse operation, the change of representation from fraction to decimal and vice versa and from percent to fraction or decimal and the use of pictorial representations, especially when working with halves and quarters.

In mental arithmetic in the context of problem solving, learners show difficulties in mobilizing strategies that are often used in mental arithmetic in mathematical contexts, because this type of task is not performed as frequently as in the mathematical context, or because, by itself, the inclusion of text to interpret can be an essential factor in reducing difficulties.

Activities to consolidate mental arithmetic skills in 6th grade students. The proposed activities should not be treated as a sequence of tasks to be followed, but as a set of resources to be used in a systematic and interrelated way.

as a set of resources to be used systematically and interlinked. Thus, they should be alternated with each other without a definite order, and the degree of difficulty of the activities can be increased. The teacher, knowing his students, will be able to elaborate other activities and articulate them.

The systematic realization of these activities helps the memorization of basic numerical facts that are essential tools for the development of calculus.

Training oral calculation can lead learners to take ownership of these facts and the construction of future strategies.

1. The game of guessing a number

In this game, the teacher exposes "guesses" that the learners must answer.

These "Guesses" require appealing to the relationship between addition and subtraction given two elements of a sum, you will have to determine the third, for example, a first guess could be:

- Think of a number, add 50 to it, and I get 70. What is the number I thought of?
- I think of a number, subtract 200, and I get 700.
- To the number 300 I add another number and get 1000. What number did I add?
- I subtract a number from the number 6000 and get 2000. What number did I subtract?
- I think of a number, add 100 and get 450. What number did I think of?
- I think of a number, add 3000 and get 8000. What number did I think of?
- I think of a number, subtract 900 and get 100. What number did I think of?

a) Estimate

b) Answer, without making the exact calculation

c) a) 235 + 185. Will it be greater or less than 500?

d) b) 567 - 203. Will it be greater or less than 300?

e) c) 418 + 283. Will it be greater or less than 600?

f) d) 639 - 278. Will it be greater or less than 400?

g) 1. For each of the following calculations, three choices are given. One of them corresponds to the correct result. Without doing the math, analyze the choices and mark which one seems to you to be the correct result:

h) 235 + 185

--- 620 ---- 320 ---- 420

i) 567 – 203

---- 464 ----- 264 ----- 364

j) 186 + 238

---- 424 ---- 224 ---- 324

k) 639 – 278

---- 361 ----- 461 ----- 261

2. How much must be subtracted from 1000 to get 755? This question could be answered by appealing to the subtraction algorithm.

Through mental calculation strategies, it could be solved in several ways. Some possibilities are:

- Calculate the complement of 755 to 1000 in different ways, relying on round numbers:

755 + 5 = 760

760 + 40 = 800

800 + 200 = 1.000

200 + 40 + 5 = 245

- Subtract successive numbers from 1 000 until you reach 755:

1000 – 200 = 800

800 – 45 = 755

200 45 = 245

3. Complete the following calculations:

a) 530 +... = 600

b) 720 +... = 1000

c) 45 +... = 1000

d) 890 +... = 3000

e) 600 + 800 =...

201. Considering that 120 x 30 = 3 600, calculate the results of:

a) 220 x 30 =

b) 320 x 30 =

c) 420 x 30 =

For each case, explain how you thought about it.

201 Solve the following problems:

(a) Rosa and Santa are sisters. They both own 45000 cup. Rosa asked for half of the money to buy a cell phone. The cell phone she wants costs 26500 cup. Will her money be enough? Why?

b) Roberto is organizing a birthday party; he called his friends and asked them for a contribution of 1500 cup for each of them. They confirmed the presence of 68 people and gave the money Roberto asked for. How many cups did Roberto collect for the party?

Conclusions

The systematization of the theories underlying the teaching - learning process of mental calculation, was selected through the importance and the reason for choosing the research object that highlighted the changes that appeared over time in the conception of the theories on mental calculation, the arguments that favored its importance, teaching - learning procedure from the strategies to acquire skills in it through the application of the methods and techniques applied. The diagnosis of the current state of mental arithmetic skills reflects exactly what was observed in the behavior of the learners when performing activities involving skills in exercises of a different nature when compiling the information from the diagnosis applied to the learners and the interview with the teacher. The responses provided the basis for presenting the activities that emerged to develop the required skills.

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