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Los ejercicios didácticos para la enseñanza-aprendizaje de la Matemática

Didactic exercises for mathematical teaching and learning

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Resumen

En este artículo se pretende fundamentar que los ejercicios matemáticos constituyen recursos didácticos para apoyar el proceso de enseñanza-aprendizaje de la Matemática, con el propósito de estimularlo como parte del proceso de la comunicación, en un contexto instructivo-educativo y como medio de aplicación de los conocimientos para consolidarlos y al mismo tiempo, sentar las bases para la adquisición de otros nuevos. Se ejemplifica la utilización de ejercicios en cada una de las funciones didácticas que convendría tener presente en el proceso de enseñanza-aprendizaje de la Matemática en particular con el contenido sobre las razones trigonométricas.

Palabras clave: Ejercicios matemáticos; Recursos didácticos; Funciones didácticas; Proceso de enseñanza-aprendizaje

Abstract

The purpose of this article is to argue that mathematical exercises constitute didactic resources to support the teaching-learning process of Mathematics, with the purpose of stimulating it as part of the communication process, in an instructive-educational context and as a means of application of knowledge to consolidate it and at the same time, to lay the foundations for the acquisition of new knowledge. The use of exercises is exemplified in each of the didactic functions that should be taken into account in the teaching-learning process of Mathematics, in particular with the content on trigonometric ratios.

Keywords: Mathematical exercises; Didactic resources; Didactic functions; Teaching-learning process.

Introduction

Mathematics contributes extraordinarily to the optimization of productive processes and penetrates, more and more, in almost all social domains. This phenomenon, called the mathematization of social life, gives an important meaning to the mathematical formation of contemporary human beings as an integral part of their personality.

In the field of intellectual development of students, the objectives express the contribution that the teaching of mathematics must make to the development of those intellectual capacities, ways of working and reasoning, thus work habits, which are essential for mathematical activity.

In order to develop thinking in general, it is necessary to carry out a constant intellectual activity that requires analyzing, synthesizing, generalizing, particularizing, abstracting, deducing, comparing and ways of working for the development of mathematical thinking, which requires the systematic training of these operations.

Exercises can be considered as any human activity aimed at developing or preserving a psychic faculty or quality, so that in the learning of the different disciplines, they serve as a complement and verification of the theoretical teaching.

The concept of exercise and their usefulness in the teaching-learning process of Mathematics have been addressed by various specialists, among which stand out: Jung (1981), Müller (1987), Ballester, Santana, Hernández, Cruz, Arango, García (1992), Ballester & Jon, (2011). According to Ballester et al. (1992), the concept of exercise in the teaching of mathematics has been defined by several specialists, most of whom agree in defining it as: "[...] a requirement for the performance of actions, solution of situations, deduction of relationships, calculation" (p. 406).

Müller (1987) defines exercises in mathematics teaching as a requirement to act, characterized by three elements:

- The objective of the actions: resides in transforming an initial situation (the given or known) into a final situation (the sought or unknown).

- The content of the actions: may be given by the elements of the mathematical subject to which they relate (concepts, propositions, algorithms) and by the type of action (identify, compare, order, classify, substantiate and control).
- The conditions of the actions: reside in the demands that the exercise poses, expressed by the degree of difficulty of the exercise. (p. 6)

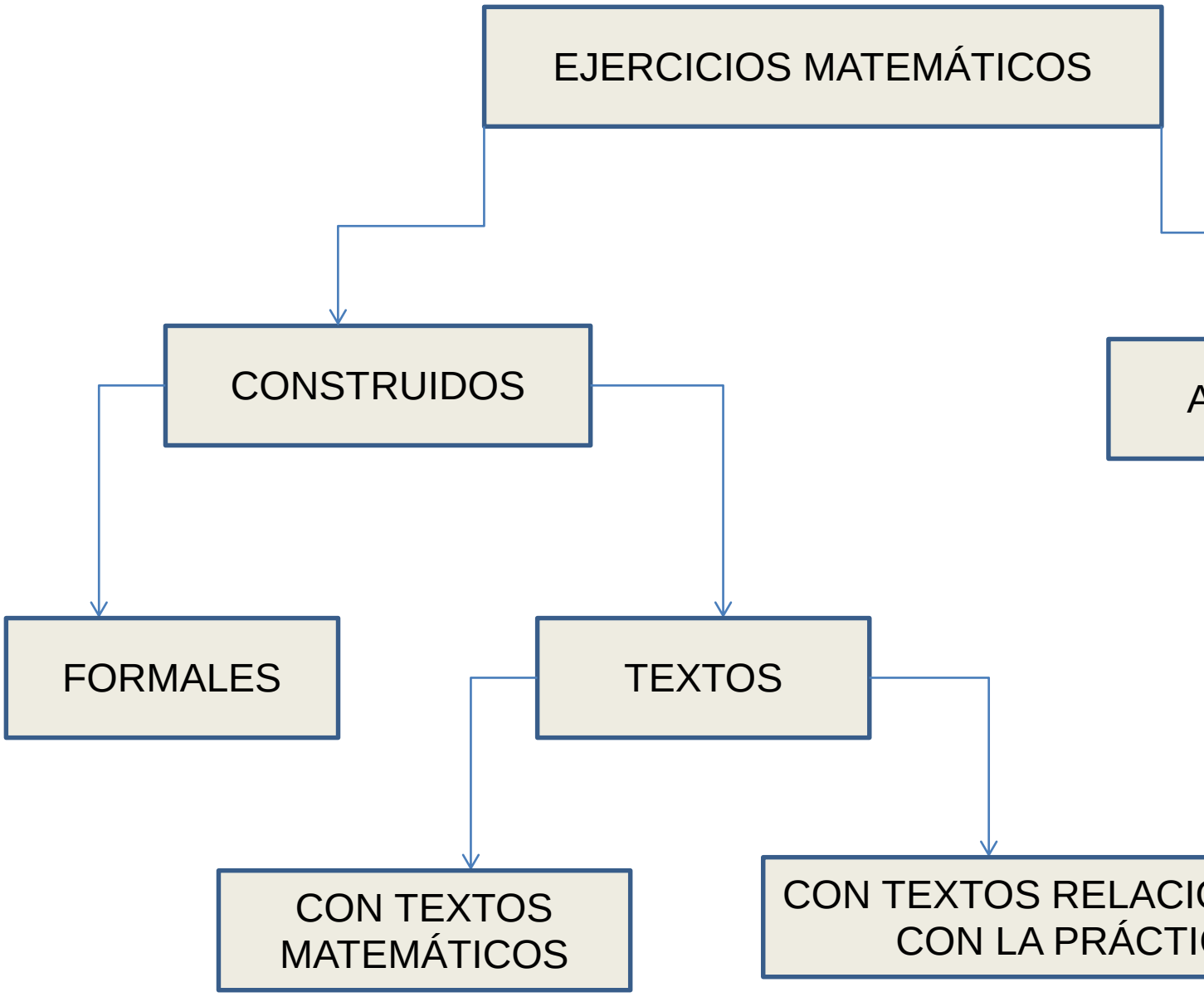
Ballester, et al. (1992) point out that an exercise is:

> A requirement that propitiates the realization of actions (...) each action must specify the objective that moves us to transform the premise to obtain the thesis; the content that includes the types of actions (identify, compare, classify, classify, substantiate, etc.), the object of the actions (concepts, propositions, algorithmic procedures), the correspondence between situations and heuristic procedures and auxiliary heuristic means. (p. 459)

There are also multiple classifications of the types of exercises. For example, Jung (1981), elaborated a classification of exercises based on the degree of abstraction in the reflection of elements and relationships, thus the type of reflection that is performed. He assumed mathematical exercises as a reference, subdividing it into two subordinate concepts:

- **Exercises of application**, those that have their origin in practice.
- **Constructed exercises**", these in turn undergo another division; the formal ones, within them he mentions (solving an equation, solving a system of equations) and on the other hand the exercises with texts made up of those whose text is purely mathematical or are related to practice.

In the textbook Mathematics sixth grade of the primary education level, a system of exercises is used following this classification.



Source: Jung, W. (1981)

In the Pedagogy field and especially in Didactics as a pedagogical discipline that investigates and elaborates the most general principles and whose object of study is the teaching-learning process, we find ourselves then with a first problem: Can exercises be considered as didactic resources? Undoubtedly, it is an aspect to reflect on that we will address later, from this perspective.

Chancusig (2017) states that didactic resources are

The alternative that can be used as an alternative during the teaching-learning process in order to fulfill an objective favorably. These interactive didactic resources refer us directly to training, training and instruction for teaching, resources are tools that have utility within an educational process, making the use of a didactic resource would help the teacher to fulfill his educational function. In general, resources provide information, serve to put into practice what has been learned and sometimes these materials are a guide for our students and we can use them as motivation for their learning. (p. 12)

According to Raffino (2020), teaching resources, teaching materials or teaching aids are any type of material or technological support that facilitates the teaching-learning process. They are usually used by students in pedagogical or training institutions as a way of complementing or making their work more efficient.

There is no strict and universal concept of what is and what is not a didactic resource. Basically because anything can be one, as long as it fulfills the function of facilitating learning or adapting it to the specific needs of a certain type of student.

At present, a didactic resource is considered to be any means, whether material or conceptual, that is used to support the teaching-learning process, which is generally face-to-face, with the purpose of facilitating or stimulating learning. Examples: blackboard, overhead projector, transparencies, slides, video, computer or others, and whose essential characteristic is its incidence in the process of educational transmission and which is also conceived in accordance with learning, forming part of educational communication, and can be classified as: oral, written and audiovisual.

Didactic resources are fundamental tools that complement the teaching-learning process. A teacher who does not use didactic resources in his classes is a teacher lacking in didactics and his students will not learn in the best way.

Therefore, it is worth highlighting the importance of didactic resources in the teaching-learning process of mathematics, so it could be said that without resources there is no learning.

For example, some resources will allow significant learning, with high student participation, while others will serve more as communicative support to the teacher, or simply as reinforcement material.

García & Cruz (2014), consider that in order to create a didactic resource we must keep in mind the following functions.

1. **Motivational function:** arouses interest in the topic or subject to maintain attention during the process of studying the content. They are tangible materials that can be manipulated by the student, motivate the learning process and can be used over and over again many times for different purposes.

2. Facilitating function: They expose clear terms that place the exposure of the students, providing information. They facilitate the strengthening of the educational process in order to enhance the quality of education.

3. Orientation function: Promotes the capacity of organization and study. They are a guide for learning, since they help to organize the information we want to transmit. In this way we offer new knowledge to the student.

4. Developmental function: It favors learning processes of skills, knowledge, they help us to exercise the skills and also to develop them.

5. Evaluation function: The didactic resources allow us to evaluate the knowledge of the students at each moment, since they usually contain a series of questions on which we want the student to reflect, it performs in the student a general review of what has been learned to then evaluate in order to stimulate a deliberation on their adequate learning. (p. 17)

Types of didactic resources

Raffino (2020), considers that didactic resources can be classified as follows:

- Permanent working material. Everything that is used daily in teaching, whether to keep a record of it, illustrate what has been said or allow other types of operations.

- Informative material. Those materials in which information is contained and which are used as a source of knowledge.
- Illustrative material. All that which can be used to accompany, enhance and exemplify the content taught, whether visual, audiovisual or interactive.
- Experimental material. That which allows students to verify by means of practice and direct experimentation the knowledge imparted in class.
- Technological material. These are the electronic resources that allow the generation of contents, the massification of the same; using mainly the so-called ICT. (p.19)

Then it can be questioned, if the exercises can be used to support the learning process:

- Can they be used to support the teaching-learning process?
- Do they enhance, facilitate or stimulate the learning process?
- Are they part of the instructional-educational communication process?
- Can they be formulated in relation to the process itself in the assimilation of knowledge, habits or skills?

Es evidente que sobre la base de estos preceptos teóricos, los ejercicios son recursos didácticos siempre y cuando cumplan con estas características, entendiéndose a todo aquel ejercicio con arreglo a determinados objetivos y cuya función sirve de apoyo al proceso de aprendizaje con el propósito de estimularlo y que constituye parte del proceso de la comunicación, en un contexto instructivo-educativo y como medio de aplicación de los conocimientos para consolidarlos y al mismo tiempo, sentar las bases para la adquisición de otros nuevos.

Es precisamente desde esta perspectiva que se procura un cambio regulado en la cantidad y cualificación de los apoyos, ayudas, estrategias, vías, metodologías, acciones didácticas y recursos para la enseñanza-aprendizaje, lo que puede involucrar aspectos tan diversos como la esfera motivacional–afectiva, el manejo de los procesos de atención, los recursos de memorización analítica, la inducción del aprendizaje y los procedimientos para el manejo eficiente de la información.

Development

Exercises as didactic resources are important ways if they are exploited in all their magnitudes, but they must also comply with certain requirements, otherwise there is a risk of obtaining results inversely proportional to those expected. Therefore, a series of premises are necessary, among others, it is important to highlight the need to consider and comment on different ways of solution as a guiding basis of the activity that forces students to reflect on the knowledge applied and the ways of carrying them out and that make possible the proposals of new solutions. Besides being ready to defend and argue their points of view in correspondence with the selected method, exercises should be proposed using different ways and reformulate the more complex ones, constituting in a first instance, an alternative aimed at stimulating the development of thinking and reflection.

When using exercises as didactic resources, it is very important to take into account that they should have a scientific character, achieving the linking of theory with practice and of the concrete with the abstract. They must be systemic and accessible, which will guarantee the solidity of the knowledge.

When using the exercises as didactic resources, the particularities and motivations of the students should be considered in order to achieve an active performance in the execution process and stimulate their reflective activity. The teacher should offer precise indications to students who do not achieve a first level of resolution according to the objectives and propose simpler exercises. No less important is the formulation of exercises, since it is a task on the conditions, requirements and ways to solve them. To achieve this, it is necessary to proceed gradually and work with different solution alternatives.

Classification of didactic exercises

We assume the classification given by Batista (2017), on didactic exercises, since exercises in the field of Didactics can be considered as didactic resources. Therefore, it is very important to consider the objectives and functions that exercises should fulfill as didactic resources.

In the different forms of teaching organization, they are governed by their system of didactic principles and functions, precisely the latter guarantee the success of the teaching-educational process by virtue of the partial objectives that each one fulfills within the teaching-learning

process itself, since they organize the modes, actions, models or steps to be followed, influencing in a positive and organized way the student's practical activity.

As has been pointed out, each function has a specific objective within the teaching-learning process, allowing the organization in the assimilation of the teaching material.

The Didactic Functions are:

- Preparation for the new teaching subject.
- Orientation towards the objective.
- Treatment of the new subject.
- Consolidation.
- Control.

Taking into consideration the different Didactic Functions, we can establish a typology of exercises as Didactic Resources, which can be classified according to their function in the different moments of the activity. (Batista, 2017).

We will use the content on trigonometric ratios to exemplify each of the exercises according to their didactic function, taking into account that there are few significant references regarding the didactic study of trigonometric topics, such as trigonometric ratios, which seem to be preliminary topics of a procedural nature and initiation to trigonometric functions. Even more so when there are not many proposals of work on this subject supported by didactic resources.

1. **Exercises for the preparation of the new teaching subject.**

- They are all those exercises whose objective is to determine the mastery or not of premises or preconditions for the assimilation of new contents.
- The previous knowledge required for the study of Trigonometric Ratios are:
- - Angles and measures. Triangles and relationship between their sides and angles.
- - Pythagorean Theorem.

- - Numerical sets (rational and irrational).
- Solving equations of 1st and 2nd degree.

The study of trigonometry requires knowledge of angles and how they are represented mathematically. They will constantly appear in the different activities and it is necessary to be familiar with them.

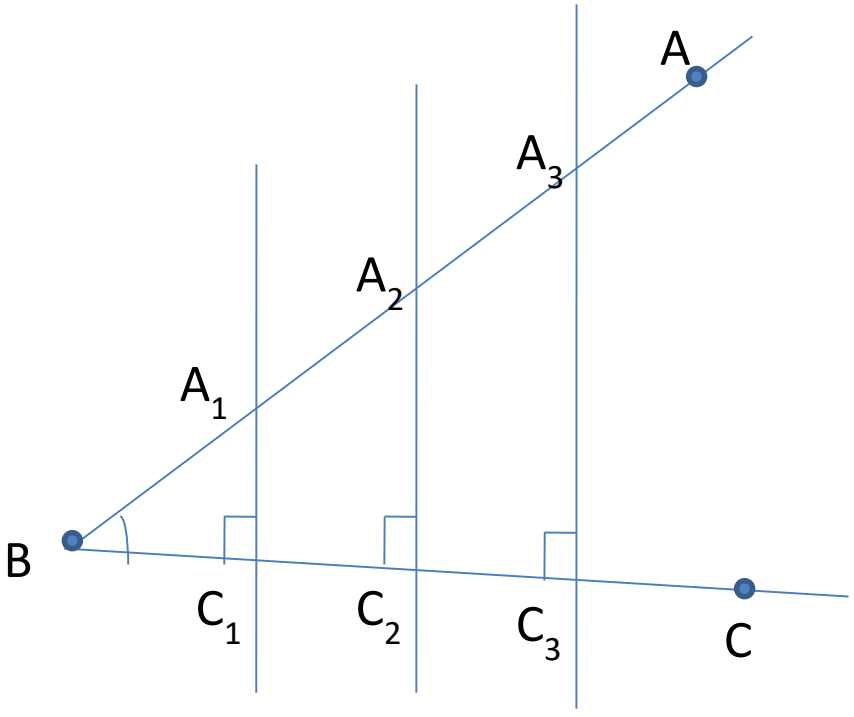
In addition, it will be necessary for students to remember the Pythagorean theorem, to use it in demonstrations and in solving some problems.

For example. To ensure the preconditions for the assimilation of trigonometric ratios in a right triangle, the following exercise can be used as a starting point.

Draw an acute angle and denote it by < ABC.

Draw three straight lines perpendicular to side $(BC)^{-}$ that intersect side $(AB)^{-}$.

Figure 1.



Source: Mathematics 9° grade

As can be shown the three triangles that are formed are similar because they have respectively two equal angles (the <ABC common to each triangle and the right angle).

Establish the equal ratios between the leg opposite the common angle <ABC and the hypotenuse in each triangle.

(A_1 C_1)/(BA_1) = (A_2 C_2)/(BA_2) = (A_3 C_3)/(BA_3)= K1

Set the equal ratios between the other leg (adjacent to <ABC) and the hypotenuse in each triangle.

(BC_1)/(BA_1) = (BC_2)/(BA_2) = (BC_3)/(BA_3) = K2

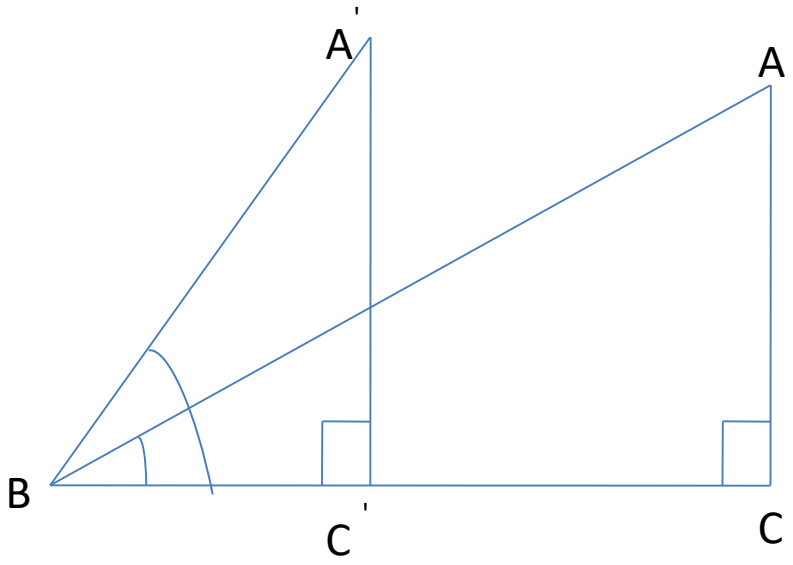
(e) Establish the equal ratios between the leg opposite the common angle <ABC and the other leg (adjacent to <ABC) in each triangle.

(A_1 C_1)/(BC_1) = (A_2 C_2)/(BC_2) = (A_3 C_3)/(BC_3) = K3

It can be seen that for the same acute angle ABC the ratio between two sides of one of the formed right triangles and the ratios of their homologous sides in the others are equal. Therefore, their result does not depend on the lengths of the sides of the triangles.

However, if we vary the amplitude of <ABC+ and perform the above analysis, we find that the ratios considered are unequal.

Figure 2.



Source: Mathematics 9° grade

Indeed, if the amplitudes of the angles A' B C and ABC are different then the triangles A' B C ' and ABC are not similar and the considered ratios are unequal.

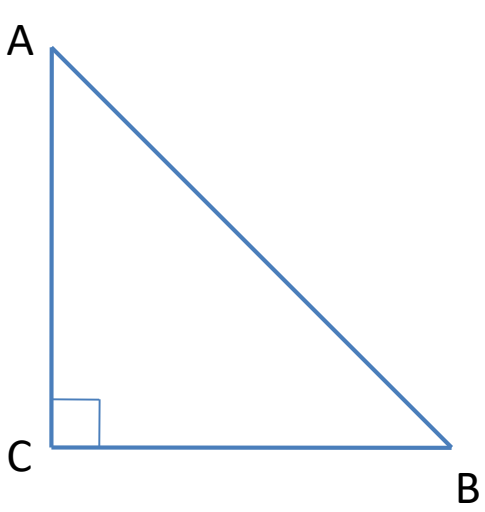
1. Goal-oriented exercises

The expectation in this type of exercise is to direct the students' attention to the object of knowledge, what they are going to know, how they are going to do it and the importance of the knowledge of the new. It is evident that if this type of exercise is done with the utmost rigor, it should arouse interest in the student toward the object of study.

For example.

We could start the motivation by analyzing that in a right triangle ABC, one side can be determined knowing the other two, if the Pythagorean theorem is applied.

Figure 3.



$AB^2 = AC^2 + BC^2$

$< A + < B + < C = 180°$

Source: Self made

Likewise, given two angles of any triangle, the third angle is calculated using the property of the sum of the interior angles of a triangle.

By analogy, it is concluded that given two elements of a right triangle we can determine the third. Now if we vary the conditions, we find ourselves with the following problem: in a right triangle, could we find the length of a side knowing another side and an acute angle, or could we calculate the amplitude of an angle knowing two sides?

a) From these questions we orient the study of trigonometric ratios in a right triangle. Trigonometric ratios are instruments that allow us to solve problems of calculating distances between two points in conditions where other methods of measurement cannot be used, because we find that one or both of the points are inaccessible or because there is an insurmountable obstacle between them.

b) We could also use the following situations to motivate and guide the study of trigonometric ratios. Related to our reality, that awakens interest in students.

c) The heights (relative to sea level) at which two points A and B are located are, respectively, 812 m and 1020m. From point A point B is seen at an angle of 30° with the horizontal plane, as shown in the figure. Determine the distance between A and B.

c) Determine the height of a building.

d) The width of a river.

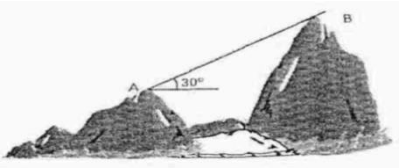


Figure 4.

Source: Self made

3. Exercises for the treatment of new material.

These are the exercises whose aim is to condition the discovery by students of the essential characteristics of concepts, judgments and the mechanisms of acquisition of new ones. Following the previous exercise that serves as a preparation for the assimilation of the content on trigonometric ratios, we can treat this subject in the following way. Elaborating the definitions of the trigonometric ratios of an acute angle in a right triangle and knowing that:

Ratio in mathematics means a division, and metric means measure, therefore the phrase trigonometric ratios can be associated to the division of the measures of the sides of a triangle.

We can resume the ratios established according to figure 1:

$$\frac{A_1C_1}{BA_1} = \frac{A_2C_2}{BA_2} = \frac{A_3C_3}{BA_3} = K_1$$

$$\frac{BC_1}{BA_1} = \frac{BC_2}{BA_2} = \frac{BC_3}{BA_3} = K_2$$

$$\frac{A_1C_1}{BC_1} = \frac{A_2C_2}{BC_2} = \frac{A_3C_3}{BC_3} = K_3$$

Definir a:

K1 = Sine of angle ABC

K2 = Cosine of angle ABC

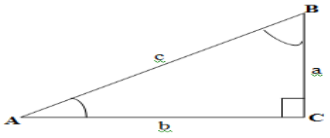
K3 = Tangent of angle ABC

To designate the sine, cosine and tangent of angle B, we will use the following notation: sin B, cos B and tag B respectively. The following exercise is then proposed to the students.

In the right triangle ACB in Figure 5:

- Identify the opposite cathetus, the cathetus adjacent to angle A and the opposite cathetus, the cathetus adjacent to angle B
- Establish the trigonometric ratios for each acute angle.

Figure 5.



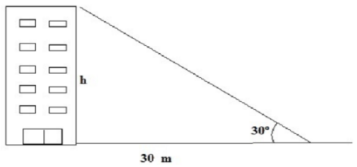
Source: Self made

3. Consolidation exercises

The objective of this type of exercise is to concretize the product of its generalizations in particular cases, having to reconsider the essential characteristics previously assimilated.

- How to calculate the height of the Victory Day School Center, if a student stands 30 m away from the base of the school and observes the height of the building at an angle of 30°.

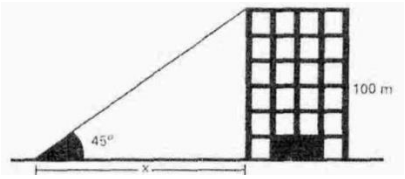
Figure 6.



Source: Self made

- When the sun is at 45° above the horizon, a building 100 m high casts a shadow of measure x. What is the result of x in meters?

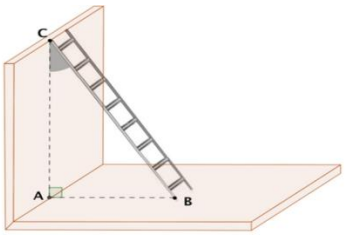
Figure 7.



Source: Self made

- A 12 meter long ladder (BC) is supported on top of a wall. Knowing that the distance from the wall to the foot of the ladder is 6 meters (AB), determine the angle of inclination of the ladder in relation to the wall.

Figure 8.



Source: Self made

5. Control exercises

It is any exercise whose objective is to determine the degree to which the object of knowledge has been assimilated. There is no better evidence for these types of exercises as didactic resources, although there are many others, than those used in exams, tests or quizzes, which allow controlling the achievement of the proposed objectives.

As can be seen in each type of exercise as a didactic resource in the classification proposed above, the essential or distinctive characteristics are the objectives or functions they fulfill in the different moments of the teaching-learning process. In all types, they are part of the educational communication, support the learning process with the purpose of stimulating it and are means of applying knowledge to consolidate it and lay the foundations for the acquisition of new knowledge.

Conclusions

Therefore, the development of exercises as didactic resources involves eliminating improvisation, anarchism and unnecessary repetition, since theory and practice, abstract and concrete, systematicity and accessibility must be scientifically linked to ensure the achievement of the objectives set.

If the classification of exercises as didactic resources is instrumented based on the function in the different moments of the class, it provides a safe and effective way to achieve learning success and the development of skills in the resolution of the problems presented in each of them.

With the proposal of the previous classification I do not intend to exhaust and much less to put an end to the problem, but to stimulate the debate thus contributing to perfect this problematic that up to the moment does not augur a quick consensus. It remains open to debate, reflection, criticism and, why not, also to suggestions.

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