

***Materiales didácticos para el proceso de enseñanza-  
aprendizaje de los sistemas de ecuaciones lineales***  
***Teaching materials for the teaching-learning process of  
the systems of linear equations***

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## **Resumen**

La investigación fue realizada en el grupo de noveno grado del Centro Mixto “Luz Palomares García” del municipio Baracoa. En las revisiones de informes de visitas y las comprobaciones se constata, que existen insuficiencias en el proceso de enseñanza-aprendizaje de los sistemas de ecuaciones lineales de dos ecuaciones con dos incógnitas. La propuesta de materiales didácticos está compuesta por un “Juego Matemático” y un conjunto de Softareas, con los que se pretende favorecer el proceso de enseñanza-aprendizaje y potenciar el uso racional de los medios y en general el de las Tecnologías de la Información y las Comunicaciones.

**Palabras Clave:** Sistemas de ecuaciones lineales; Materiales didácticos; Juego Matemático; Conjunto de softareas

## **Abstract**

The research was carried out in the ninth grade group of the “Luz Palomares García” Mixed Center of the Baracoa municipality. In the review of visit reports and the verifications it is confirmed that there are inadequacies in the teaching-learning process of the systems of linear equations of two equations with two unknowns. The proposal of didactic materials is composed of a “Mathematical Game” and a set of Softareas, with which it is intended to favor the teaching-learning process and

promote the rational use of the media and in general that of Information Technology and the Communications.

**Keywords:** Systems of linear equations; Teaching materials; Mathematical Game; Softtasks set

## Introduction

As a consequence of the Scientific and Technological Revolution initiated in the 20th century, mediated by the development of information technology and communications and the undeniable process of globalization, science and technology increasingly occupy a fundamental place in the productive system. and in everyday life in general. In this regard Salazar, (2004) in his article "Interdisciplinarity as a trend in science education" states:

The New Information Technologies (NTI) facilitate the search for new knowledge in a creative way, due to the possibility they offer of not having to follow the usual linearity in the texts. From an educational point of view, they contribute to the transformation of the personality of the students, allowing them to prepare themselves more broadly in their profession, optimize their time and enter modern information search systems. (p .54)

In the 21st century, Cuban education continues to be committed to this social project and continues to enrich it in order to guarantee the sustainable and integral human development of the new generation, and that Cuba's future depends on what it is capable of doing in this Third Educational Revolution. Our Commander in Chief, in his speech of September 16, 2002 states:

Today it is about perfecting the work done and starting with entirely new ideas and concepts. Today we are looking for what we believe should be and will be an educational system that corresponds more and more to equality, full justice, self-esteem and the moral and social needs of citizens in the model of society that the people of Cuba have proposed to create. (2002).

In the teaching of Mathematics there are different trends that appear in the book "Comparative Study in the training of teachers of Basic Secondary School" in their article "Mathematics: their teaching and learning", according to Godino, Llivina, Arancibia, but for the reason of the proposal we refer to those of Miguel de Guzmán:

- Mathematics education as a process of "inculturation".
- Mathematics education with continuous support in the direct intuition of the concrete with

permanent support in the real.

- The center of Mathematics education is the processes of Mathematics education.
- The use of new technologies in the teaching of Mathematics.
- Mathematics teaching based on motivation (Guzmán, 2007, p.25)

In spite of everything raised in the Mathematics teaching-learning process there are inadequacies and the academic results of the students in solving the systems of linear equations of two equations with two unknowns are low, being an unsolved problem in the Basic Secondary. When this occurs, the promotion becomes an indicator of insufficient strategies and pedagogical alternatives that affect the high rates of school failure. The “Luz Palomares García” Mixed Center of the Baracoa municipality is not oblivious to what was stated above, since its greatest difficulties lie in:

- Traditional classes with a reproductive, passive and poorly individualized learning.
- Insufficient motivation.
- Insufficient use of the potential of educational software.

The analysis of these difficulties served as a fundamental motivation to convert the following solution into the purpose of this work:

Scientific Problem: How to favor the teaching-learning process of the systems of linear equations of two equations with two unknowns in the ninth grade of the “Luz Palomares García” Mixed Center of the Baracoa Municipality?

## **Development**

Currently, the systems of linear equations of two equations with two variables and the systems of three linear equations with three variables are studied, as well as the problems that are modeled through those systems, as proposed in the Guideline of Mathematics “Work with variables, equations, inequations and systems of equations and inequations” in the Ninth Grade Mathematics Program, which describes:

In the Basic Secondary school, the mathematical knowledge and skills on the work are consolidated and systematized with variables that the elementary school students possess and the elements of the algebraic technicality are studied, which allow their use to be expanded, as they are used as symbols

with the which is operated. From sixth grade they are able to solve problems that lead to linear equations; in eighth, they propose more complex linear equations and systems of linear equations to model given situations and in ninth grade, they can do the same by developing procedures that allow them to determine the real solutions of quadratic equations.

Vega, referring to teaching in the solution of equations in secondary school, introduces the problem as an "equation", which we must solve: Teaching + X = Learning. In other words, teaching is not equal to learning, something is needed, an unknown term. X = methods, teaching strategies, games etc. The only thing we are sure of is that we will not achieve it with the "same old class". (Vega, 2016, p.2). Criteria we share. According to Cabero (2001), there is a diversity of terms to define the concept of teaching materials, such as:

- Means, medium.
- Auxiliary means.
- Didactic resources.
- Audiovisual medium.
- Materials.

This diversity of terms leads to a problem of definition of the concept, as well as to the extent to which they are considered. (Cabero, 2001).

That is, each author gives a specific meaning to the concept, which leads to a much broader picture in terms of teaching materials.

The terminology used to name the teaching materials leads to consideration, according to Cebrián (p. 290). Other authors such as G. García (1984), González (1986), Rodríguez (2000), Fernández (Pedagogy 2005) refer to the definition of teaching means, so the author considers that in one way or another they converge in that it is a support material in the teaching-learning process and assumes Solano's definition of teaching materials for what is suggested in the proposal.

The television, the video and the software are efficient auxiliary means for the teacher in the preparation of classes since it contributes to a rationalization and optimization of the activities of the teacher and that of the students.

Animated representations are thus facilitated, showing complex processes impossible to observe and understand that are characteristic of the subject because it is abstract (Zamora, 2008, p.41).

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The didactic materials that are proposed are a Mathematical Game and a set formed by 8 Softasks:

**Game:**

**Title: "Mathematical game"**

**Objective:** Solve exercises or problems related to the systems of two linear equations with two variables.

**Structure:** 2 dice, 4 chips, a set of cards and flat support.

**Illustration:** The dice and chips used are the traditional ones. The cards contain exercises and problems related to the systems of linear equations of two equations with two unknowns. The flat support is made up of different boxes: the starting one (start of the game), the ones that travel to the finish line and some of them that indicate returning, advancing or waiting for a turn and the finishing one (end of the game).

**Organization:** 4 teams will be formed whose names correspond to the colors of the chips and from each one a representative will be chosen who will be in charge of playing. The rest will be pending to answer the card question, if necessary. The teams will be formed; balanced, by students of each level of performance.

The teacher's role is as a counselor, moderator, also controls and evaluates the process.

**Procedure:** It should be started by rolling the dice for each representative so that whoever draws the highest number on the dice is the first to play, when throwing the dice the number obtained will indicate the amount of boxes to advance, to do so, they must choose a card and answer the question that appears on it (can be done by the student or any of his team); if it is not be done correctly, the opportunity to move forward will be losen , leaving the way clear for another team to respond and advance the boxes. The winning team will be the first to reach the goal.

**Evaluation:** It will be taken into account to evaluate the student: the self-evaluation, the co-evaluation and evaluation of the work of the teams have to be profound, considering the final results, but above all the process and the individual and group advances. The compliance with the elementary rules for group work and learning and creativity indicators must be taken into account.

The flexibility and adequacy of the research activities in the software according to the characteristics of the student is insisted.

**Suggestions to implement it:** Due to the characteristics of the school, the established machine time is alternated with other activities, so that would be a space to introduce this game and in the Mathematics Knowledge Encounters.

**Softask 1:**

**Introduction:**In previous classes you studied the position relationship between two lines. With a view to remembering this, it is proposed that you find the solution for the following exercise:

Task formulation: How are the system solutions?

$$y = 3x + 8$$

$$y = 3x - 2$$

*No solution*

*Infinite*

*A solution*

*The solution is  $x = 0$  and  $y = 8$*

Suggestion on how to proceed: The student must solve the exercise in his notebook and select the correct answer in the educational software Mathematical Elements of the El Navegante Collection in the Exercises Module; Unit 3: The variables, exercise 167. The teacher must take advantage of the student's result to enhance the geometric meaning and to systematize knowledge, through the following questions: What does it mean that the system of equations has no solution? Give at least three examples of the environment around you that satisfies that condition. How did you solve it? Based on the discussion of the resolution of the exercise, the following question is proposed: What is the most advantageous way to solve it?

**Resources:** Navegante Collection (Basic Secondary Education),

Educational software: "Mathematical Elements" (In the form of a Web page)

**You can help yourself:**

**Bibliography:** From the eighth grade textbook, Chapter 4, 1991 Edition.

Indications for the evaluation of the task: They will be carried out by teams taking into account the levels of performance; while the student is told which way to choose to solve the system, according to performance achieved. The teacher will discuss with each student orally and in writing the work that corresponds to him, and then with the team.

Forms of evaluation: the indicators that will be taken into account in the qualification: performance and independence. An evaluation will be issued according to the indicators and the student's individual and collective result.

### **Softasks 2:**

Introduction: In how many possible ways can you solve the systems of linear equations of two equations with two unknowns? Solve it in your notebook by each of the ways found. The solution will be the same by any means. What do you believe?

### **The following exercise is proposed:**

**Formulation of the task:** Solve the system of equations by the substitution method:

$$2x - 3y = -13$$

$$4x + 2y = -2$$

### **Suggestion on how to proceed:**

Previously the teacher tells you to perform the exercise by the method of addition-subtraction. Compare the result you obtained with the one that appears in the educational software Mathematical Elements of the El Navegante Collection in the Content Module;

Topic 3: The variables; section 3.3 Equalities containing variables; its heading 3.3.4 Systems of linear equations, p.4.

Resources: Navegante Collection (Basic Secondary Education), Educational software: "Mathematical Elements" (In the form of a Web page)

### **You can help yourself:**

Bibliography: From the eighth grade textbook, Chapter 4, Edition 1991. So you can remember the ways to solve systems of equations of two linear equations with two variables. Indications for the evaluation of the task: They will be carried out by teams taking into account the levels of

performance; while the student is told which way to choose to solve the system, according to performance achieved. The teacher will discuss with each student orally and in writing the work that corresponds to him, and then with the team. Forms of evaluation: the indicators that will be taken into account in the qualification: performance and independence. An evaluation will be issued according to the indicators and the student's individual and collective result.

### Softasks 3:

Introduction: Find, if possible, the ordered pair that satisfies the following system of equations.

Task formulation: Obtain the solution of the following system of equations:

$$-x - y = -5,7$$

$$x - 3,3 = y$$

$$S = \{ ( \quad , \quad ) \}$$

Suggestion on how to proceed: The student must copy and solve the exercise in his notebook and enter the answer in the educational software Mathematical Elements of the El Navegante Collection in the Exercises Module; Unit 3: The variables, exercise 190. In the end, check if the answer is correct or not. On the basis of the discussion of the resolution of the exercise the following question is proposed: How else can I solve this exercise. What is the most advantageous way to solve it? It works with rational numbers to enhance the calculation; deteriorated element in the students. Resources: Navegante Collection (Basic Secondary Education), Educational software: "Mathematical Elements" (In the form of a Web page)

You can help yourself:

Bibliography: From the eighth grade textbook, Chapter 4, 1991 Edition.

Indications for the evaluation of the task: They will be carried out by teams taking into account the levels of performance; while the student is told which way to choose to solve the system, according to performance achieved. The teacher will discuss with each student orally and in writing the work that corresponds to him, and then with the team. Forms of evaluation: the indicators that will be taken into account in the qualification: performance and independence. An evaluation will be issued according to the indicators and the student's individual and collective result.

### Softask 4:



Introduction: Find, if possible, the ordered pair that satisfies the following system of equations. Task formulation:

Obtain the solution of the following system of equations:

$$\frac{3}{4}x + \frac{5}{3}y = 8$$

$$\frac{x}{2} + 1 = y$$

S= [(     ,     ) ]

Suggestion on how to proceed: The student must copy and solve the exercise in his notebook and enter the answer in the educational software. Mathematical Elements of the El Navegante Collection in the Exercises Module; Unit 3: The variables, exercise 192. In the end, check if the answer is correct or not. Based on the discussion of the resolution of the exercise, the following question is proposed: How else can I solve this exercise? What is the most advantageous way to solve it? It works with rational numbers to enhance the calculation; deteriorated element in students and work individual differences. Resources: Navegante Collection (Basic Secondary Education), Educational software: “Mathematical Elements” (In the form of a Web page)

**You can help yourself:**

**Bibliography:** From the eighth grade textbook, Chapter 4, 1991 Edition.

Indications for the evaluation of the task: They will be carried out by teams taking into account the levels of performance; while the student is told which way to choose to solve the system, according to performance achieved. The teacher will discuss with each student orally and in writing the work that corresponds to him, and then with the team. Forms of evaluation: the indicators that will be taken into account in the qualification: performance and independence. An evaluation will be issued according to the indicators and the student's individual and collective result.

**Softask 5:**

**Introduction:** With a view to answering the following questions that students usually ask.

Where do I use knowledge about the systems of linear equations of two equations with two unknowns? What is the use of knowledge about the systems of linear equations of two equations

with two unknowns? In how many possible ways can I solve systems of linear equations of two equations with two unknowns? What geometric meaning do the solutions of the systems of linear equations of two equations with two unknowns have?

The following exercise is proposed:

Formulation of the task: At a traffic control point, expressions that describe the trajectories of two rolling objects that move on a track with very low visibility are known following the equations:

$$x + y = 5$$

$$2x - 4y = -2$$

The traffic control point personnel, based on the assumption that they will maintain that trajectory, must determine the possible collision points, that is, the intersection points of the lines that describe their trajectories and identify those lines. Answer in your notebook. Suggestion on how to proceed: This is an exercise taken from the Prematic software and was reworked by the tutor and the author of this work whose objective is to introduce the problems that lead to systems of linear equations. Compare the result you obtained with that of the other classmates. Check where you were wrong? How did you proceed unlike your classmates? What is the easiest solution?

**Resources:** Educational software: Prematic (In the form of a Web page).

**You can help yourself:**

**Bibliography:** From the eighth grade textbook, Chapter 4, 1991 Edition.

- So you can remember the ways to solve systems of equations of two linear equations with two variables.
- Compare the result obtained.

Indications for the evaluation of the task: They will be carried out by teams taking into account the levels of performance; while the student is told which way to choose to solve the system, according to performance achieved.

The teacher will discuss with each student orally and in writing the work that corresponds to him, and then with the team. Forms of evaluation: the indicators that will be taken into account in the

qualification: performance and independence. An evaluation will be issued according to the indicators and the student's individual and collective result.

This last Softask allows us to introduce three problems that lead to the solution of the systems of equations of two with two that appear in the software; "Reworking" the previous exercise so that the modeling systems coincide with the system proposed in the following exercises; in which they have to compare the result with that previously made in their notebooks; that is, they are the same data in different problem situations.

### **Teaching materials "Softask"**

#### **Softasks: 6, 7 and 8**

**Softasks 6:** Exercise 170. In a gymnastic table of 196 participants 6 circles and 4 stars are formed. For a circle and a star, 40 students are needed. The star is formed with:

\_\_\_\_\_ 20 students

\_\_\_\_\_ 18 students

\_\_\_\_\_ 30 students

**Softask 7:** Exercise 171. In Cuba there are 714 health facilities between polyclinics and hospitals, half of the polyclinics exceeds 153 to 25% of the hospitals. How many hospitals and polyclinics are there in Cuba?

**Softask 8:** Exercise 176. Two students voluntarily perform the planting of 536 tree positions. Student A planted 25 positions less than double the amount of posture sown by student B. How many positions do each student sow?

**Methodological Suggestions:** Before solving problems, we recommend taking into account the General Heuristic Program.

## **Conclusions**

The systematization of the theoretical references that was carried out facilitated the integration of the philosophical, psychological, didactic foundations that allowed the deepening of the content on systems of linear equations of two equations with two unknowns and taking significant steps in the

effort to achieve a teaching associated to favor the preparation of the students; based on the theory of the near development zone of Vygotsky.

The elaboration of the didactic materials with their corresponding theoretical foundation made it possible to provide a support material for the teaching-learning process of the systems of linear equations of two equations with two unknowns causing the rise to higher levels, which constitutes one of the objectives of the teaching of Mathematics. Validating this proposal confirms its effectiveness; corroborated in the educational practice and with the results of the pedagogical tests. In addition, in the different spaces of socialization of this work, which allowed us to enrich and improve it, achieving satisfactory results and those that I relate below:

- Mafis 2017 event.
- ExpoAnir at the University of Guantanamo, 2018.
- National ExpoAnir held at the University of Medical Sciences, 2018.
- Student Scientific Day, 2018.

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